Reduction of Leading-Edge Noise by Tailored Turbulence Anisotropy

Alistair D. G. Hales[1,*], **Lorna J. Ayton**[2]
[1]DAMTP, University of Cambridge, Cambridge, United Kingdom
[2]DAMTP, University of Cambridge, Cambridge, United Kingdom
*Email: adgh3@cam.ac.uk

Abstract

This paper investigates the effect of anisotropic turbulence on the generation of turbulence-aerofoil interaction noise, also known as leading-edge noise, for a rigid plate. Thin aerofoil theory is used to model an aerofoil as a semi-infinite plate and the scattering of incoming turbulence is solved via application of the Wiener-Hopf technique. This theoretical solution encapsulates the diffraction problem for gust-aerofoil interaction, and is integrated over a wavenumber-frequency spectrum to account for general incoming turbulence. The specific wavenumber-frequency spectrum in the anisotropic case can be obtained using the method of Gaussian decomposition, in which the generalized spectrum is approximated through the weighted sum of individual Gaussian eddy models.

Keywords: Leading Edge Noise, Noise Control, Aeroacoustics, Turbulence Modelling

1 Introduction

Leading-edge noise, also known as turbulence-aerofoil interaction noise, is produced by the scattering of surface pressure fluctuations that are due to turbulent velocity fluctuations of a given incoming flow by the leading edge of a blade. It is known to be a dominant noise mechanism for many applications, such as in wind turbines, helicopter rotors, and turbofan engines. In this latter case, for engines with multi-row rotor systems, we find that the wake from turbulence interacting with rotor blades impinges on the downstream blade and is a dominant source of noise [1]. Regarding wind turbines, we find that the interaction of the blades with atmospheric turbulence causes unwanted noise particularly at low frequencies. Thus, there are industrial and social reasons to focus on reducing leading-edge noise, particularly to counteract harmful noise pollution.

2 Body of the paper

In this paper we calculate the power spectral density denoted $\Psi(\omega)$ for the rigid leading edge diffraction problem, given by

$$\Psi(\omega) = \int_{-\pi}^{\pi} \int_{\mathbb{R}^3} P(\mathbf{k}, \theta) \Phi_{22}(\mathbf{k}) \delta\left(k_1 - \frac{\omega}{U_\infty}\right) \mathrm{d}\mathbf{k} d\theta,$$

which will be given in SPL as a measure of the total observed far-field noise.

We discuss the two terms in the integrand separately, since they require different mathematical tools to model and solve them accurately and efficiently. This inherent duality in the problem provides scope in noise reduction by targeting both the scattering via the plate modelling, and by targeting how the spectrum modelling distributes energy.

First, $P(\mathbf{k}, \theta)$ is the far-field pressure perturbation found by solving a convected Helmholtz equation for the scattered velocity perturbation ϕ_s and then applying Taylor's frozen turbulence hypothesis to relate this to the pressure via $p' = -\rho_0 \frac{D\phi}{Dt}$.
Having set up our governing equations, which include a pressure jump upstream and Neumann boundary conditions along the plate, we apply the Wiener-Hopf technique and the method of steepest descent to find the far-field scattered pressure.

Φ_{22} is the energy density spectrum, describing how the turbulent kinetic energy is distributed over wavenumbers. We model this using the method of Gaussian decomposition derived in [3] and applied to leading edge noise in [4].
We extend this approach to the case of axisymmetric turbulence via the model given in [2], however, unlike [4], we take a more mathematical approach. We derive a model that sums axisymmetric Gaussian filter kernels in order to approximate a generalized von Kármán type model (which differs by generalizing the inertial subrange scaling of the turbulence via the exponent

p in the denominator) of the form

$$\Phi^a_{22}(\mathbf{k}) = \frac{2k_T(L^*\Lambda_a)^5\Lambda_r^4\Gamma(p)}{(1+2u_r^2)\pi^{3/2}\Gamma\left(p-\frac{5}{2}\right)} \times \frac{k_1^2+\gamma k_3^2}{[1+L^{*2}\Lambda_a^2(k_1^2+\Lambda_r^2(k_2^2+k_3^2))]^p}.$$

Here we define the kinetic turbulence energy k_T, L^* is a lengthscale parameter that ensures

$$\Lambda_a = \int_0^\infty R_{11}(x,0,0)\mathrm{d}x.$$

where R_{11} is the $1,1$ velocity autocorrelation, or in the context of our work the spatial Fourier transform of the wavenumber spectrum $\Phi_{11}(\mathbf{k})$. The subscripts a and t refer to quantities in either the axial or transverse direction with respect to the oncoming mean flow, and r denotes that it is a ratio of transverse to axial. Moreover, we use the parameter $\gamma = 2u_r^2 - \Lambda_r^2$ from [2]. The ratios of root mean velocity and integral lengthscales in the transverse and axial direction satisfy the important constraint $2u_r^2 \geq \Lambda_r^2$ in [2], which ensures that the power-spectral density remains non-negative.

Then, by finding a suitable weighting function f we get a good approximation to this axisymmetric model. The primary benefit of this turbulence model is that we can sum individual kernels with different p values or different lengthscale ratios, which effectively allows us to adapt our modelling to the behaviour of the turbulence itself.

In figure 1, in which we have fitted our model to experimental data for a rigid plate in slightly anisotropic turbulence from our collaborators at UNSW Sydney. Different ratios of anisotropy were obtained by placing the leading edge at different distances from the wake of a cylinder. In figure two we further test the trends observed from the experimental data using our model which was verified as accurate over a wide range of frequencies. We first test fully isotropic data, and then reverse the anisotropy ratio to favour the streamwise direction. Both cases show good noise reduction.

When matching to data, we find that $p = \frac{11}{3}$ (the constant found for the Rapid Distortion Theory description of turbulence) gave excellent agreement to high frequency cut-off, verifying that cylinder turbulence has different scalings in the inertial subrange than standard grid generated turbulence that is usually modelled with either Liepmann ($p = 3$) or von Kármán ($p = \frac{17}{6}$) turbulence models. The benefit of the Gaussian decomposition model is that the turbulence can be studied without considering scattering, and the behaviour can be modelled beforehand thanks to its versatility.

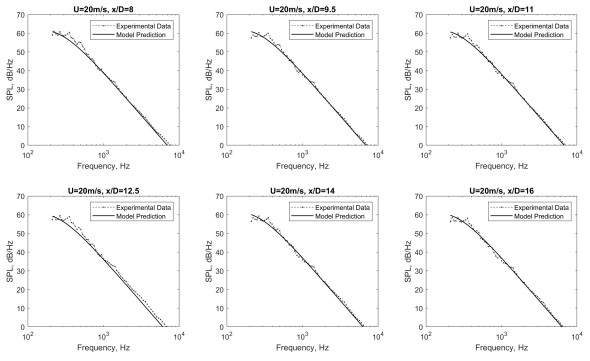

Figure 1: Comparison of model and experimental data, ratio of anisotropy varied

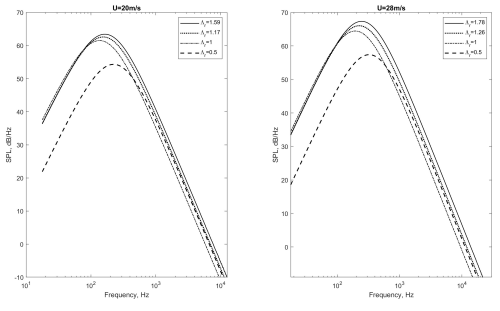

Figure 2: Model predictions for both an isotropic spectrum and a spectrum favouring the streamwise direction

3 References

References

[1] B.Lyu, L.J. Ayton and P.Chaitanya, *Journal of Sound and Vibration*, **462**, (2019) pp.114923

[2] E.J. Kerschen and P.R. Gliebe, *AIAA Journal*, **19**,6 (1981) 717–723

[3] A. Wohlbrandt, N. Hu, S. Guérin and R. Ewert, *Computers Fluids*, **132** (2016) 46–50

[4] F. Gea-Aguilera, J. Gill and x. Zhang, *Computers and Fluids*, **157** (2017) 240–252