Effective wave motion in periodic origami structures

Othman Oudghiri-Idrissi[1,*], **Bojan B. Guzina**[1]
[1]Department of Civil, Environmental & Geo- Engineering, University of Minnesota, Twin Cities, Minneapolis, MN 55405, USA.
*Email: oudgh001@umn.edu

Abstract

We establish a dynamic homogenization framework for describing linear elastic wave motion in periodic origami structures by adopting a "bar-and-hinge" modeling paradigm. In this setting, we conduct a finite wavenumber – finite frequency homogenization in the spectral neighborhood of simple, repeated, and nearby eigenfrequencies at an arbitrary wavenumber within the first Brillouin zone. For completeness, the source term representing the nodal forces is expanded in Bloch waves and included in the analysis. We express the leading-order (system of) effective equation(s) in a given spectral neighborhood, and we approximate asymptotically the corresponding Bloch dispersion relationship. We illustrate the proposed framework by (i) comparing numerically the Bloch dispersion relationship to its asymptotic approximation for a 2D-periodic Miura-ori structure and (ii) computing the effective motion near an isolated eigenfrequency.

Keywords: Dynamic homogenization, periodic origami structures, tunable metamaterials

1 Origami structures

Origami, the ancient Japanese art of paper folding, has become a source of technologic inspiration and a keen subject of studies for its unique scalability, programability, deployability and configurability properties. This class of structures have rapidly found applications ranging from nano to large scales in science and engineering with examples including medical stents, energy absorbing structures, vibration control instruments, emergency shelters, inflatable structures, and large spacecraft structures. Origami-like engineered structures have also been found to exhibit the so-called metaproperties that are not observed in conventional structures and natural materials, such as auxeticity, infinite shear stiffness, negative bending stiffness, unidirectional flexibility or strain reversal (in origami tubes).

The existing literature on the homogenization of periodic origami-like structures can be classified chiefly by (i) the dynamic regime of study, and (ii) the mechanistic framework adopted to model the origami panels. In the static regime, Lebee and Sab [4] pursued a homogenization analysis of periodic thick plates using the bending-gradient plate theory. On the other hand, when an origami structure is modeled using a bar-and-hinge paradigm [2], the existing literature on trusses can be deployed to study its dynamic properties. In this framework, Craster et al. [1] extended the homogenization theory of discrete periodic lattices to high frequency and small wavelengths regimes. A generic finite wavelength-finite frequency (FW-FF) homogenization framework was proposed by Guzina et al. [3] to describe the effective wave motion in periodic media with rectangular Bravais lattices in the spectral neighborhood of simple, repeated, and nearby eigenfrequencies located at the origin or vertices of the first Brillouin zone (BZ). More recently, Oudghiri-Idrissi et al. [5] extended the FW-FF homogenization framework to "perforated" periodic continua supported on general Bravais lattices by considering the spectral neighborhood of an arbitrary wavenumber within the first BZ and eigenfrequency clusters of arbitrary size.

By building upon the FW-FF homogenization approach [3, 5], we aim to better understand the wave motion in origami structures by providing an origami-specific, dynamic homogenization framework that leverages the bar-and-hinge paradigm [2]. Such formulation specifically aims to: (i) capture the essential dynamics of the problem, (ii) analytically illuminate the origami behavior near spectral singularities (e.g. Dirac points), (iii) reduce the computational cost, and (iv) aid the design of programable and tunable (periodic) origami structures.

2 Effective wave motion

We model periodic origami structures using a "bar-and-hinge" paradigm where: (i) the folding of the structure and bending of the origami panels are modeled via elastic hinges, and (ii) the in-plane deformation of each panel is modeled with elastic bars [2]. Using the Bloch-wave expansion of the source term acting on the nodes of the discretized structure, the analysis is reduced, thanks to the linearity of the system and periodicity of the material properties, to that of a "unit cell" of the origami structure. The homogenous part of the unit cell system constitutes an eigenvalue problem that defines the "origamons", i.e. the Bloch waves that can propagate freely in the discrete periodic origami structure (DPOS) at a given eigenfrequency, and which form a local basis that describes the leading-order response of the DPOS. By pursuing an asymptotic analysis in the neighborhood of isolated, repeated, and nearby eigenfrequencies at the wavenumber of interest, we express the leading-order approximation of the total and effective motion in those spectral neighborhoods. In this way, we formulate the (system of) effective equation(s) that includes the homogenized source term and forms the basis from which the leading-order asymptotic approximation of the corresponding dispersion relationship is obtained. Fig. 1 illustrates a discretized unit cell of the 2D-periodic Miura-ori structure. Fig. 2 examines the performance of the homogenization framework by plotting the numerically-evaluated dispersion relationship together with its asymptotic approximation computed at different wavenumber-eigenfrequency pairs. Fig. 3 illustrates the effective wave motion in a 2D-periodic Miura-ori structure for an excitation frequency (b) inside a band gap, and (c) inside a pass band.

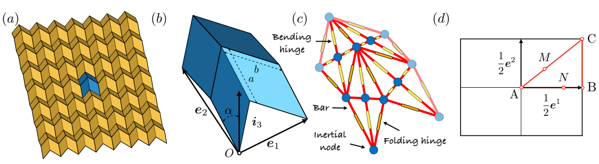

Figure 1: (a) Truncated 2D periodic Miura-ori sheet (b) unit cell of periodicity, (c) discretized unit cell, and (d) first BZ of the lattice.

References

[1] R.V. Craster, J. Kaplunov and J. Postnova (2010). High-frequency asymptotics, homogenisation and localisation for lattices. *Quart. J. Mech. Appl. Math.*, **63**, 497-519.

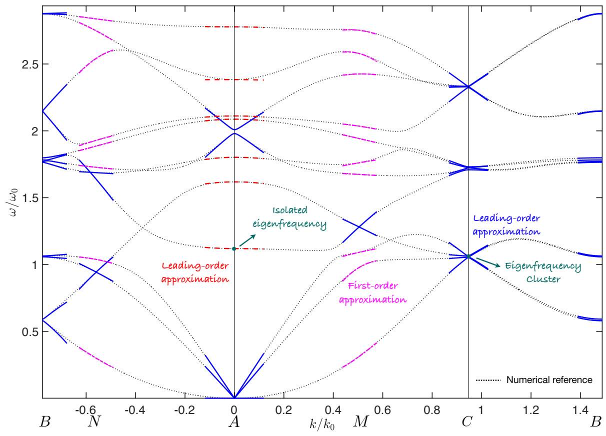

Figure 2: Approximation of the first twelve dispersion branches for the Miura-ori periodic structure.

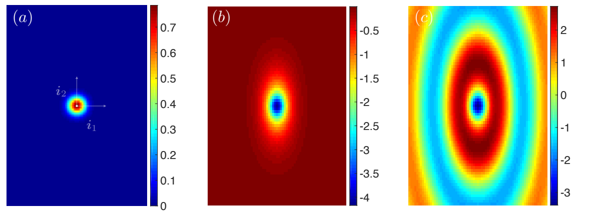

Figure 3: (a) Amplitude of the driving force and corresponding leading-order approximation of the induced effective motion within: (b) band gap, and (c) pass band near the origin of the first BZ.

[2] E.T. Filipov, K. Liu, T. Tachi, M. Schenk and G.H. Paulino (2017). Bar and hinge models for scalable analysis of origami. *Int. J. Solids Struct.*, **124**, 26-45.

[3] B. B. Guzina, S. Meng, O. Oudghiri-Idrissi (2019). A rational framework for dynamic homogenization at finite wavelengths and frequencies. *Proc. Roy. Soc. A* **475**, 20180547.

[4] A. Lebee and K. Sab (2012). Homogenization of thick periodic plates: Application of the Bending-Gradient plate theory to a folded core sandwich panel. *Int. J. Solids Struct.*, **49**, 2778-2792.

[5] O. Oudghiri-Idrissi, B.B. Guzina and S. Meng (2021). On the spectral asymptotics of waves in periodic media with Dirichlet or Neumann exclusions. *Quart. J. Mech. Appl. Math.*, **74**, 173-221.