Numerical Study on Spatial Evolution of Progressive Waves using Fully Nonlinear Numerical Wave Tank

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Abstract

A spatial evolution by nonlinear wave-wave interaction was studied using the 2D FN-NWT technique which was based on boundary element method with Rankine panels. The present results were compared with previous experimental data for spatial evolution of Bi-chromatic waves, and the difference between fully nonlinear and linear calculation results was investigated. The nonlinear spatial evolution of traveling waves by wave-wave interaction was well realized through the fully nonlinear calculation, and it was in good agreement with the experimental results. The farther the wave propagated, the greater the difference between linear and nonlinear calculations. The spatial evolution of irregular waves was numerically analyzed.

Keywords: Bi-chromatic waves, Spatial evolution, Fully nonlinear numerical wave tank

1 Introduction

In general, a rogue wave is defined as a wave which height is greater than two times the significant wave height defined as the mean of the highest one third of waves occurring over a certain time period [1]. There are several reasons for the occurrence of sudden huge waves. One typical reason is that the extreme wave may occur by nonlinear interactions from various small waves as wave group. To simulate a spatial evolution of group waves, two-dimensional fully nonlinear potential flow numerical wave tank (2D-FN-NWT) technique was used, and this numerical model is based on boundary element method with constant Rankine panels. In addition, the mixed Eulerian-Lagrangian (MEL) method was applied for updating nonlinear free surface water particles in the time domain analysis. Bi-chromatic waves and irregular waves were numerically simulated, and linear and fully nonlinear calculation results of wave evolution by wave-wave interaction were compared.

2 Mathematical formulation

The computational domain is filled with incompressible, irrotational, and inviscid fluid in the 2D FN-NWT, so the governing equation can be Laplace equation using velocity potential(ϕ) and continuity equation satisfied in the fluid domain.

$$\nabla^2\phi = 0, \tag{1}$$

$$\alpha\phi_i = \iint_\Omega (G_{ij}\frac{\partial\phi_j}{\partial n} - \phi_j\frac{\partial G_{ij}}{\partial n})ds \tag{2}$$

$$G_{ij} = -\frac{1}{2\pi}\ln R_{ij} \tag{3}$$

$$R_{ij} = \sqrt{(x_i - x_j)^2 + (z_i - z_j)^2} \tag{4}$$

where α is a solid angle and it was set to 0.5 on the boundary of computational domain. ϕ_i and ϕ_j mean the velocity potential at the source (x_i, z_i) and field (x_j, z_j) points, respectively. For the free surface boundary condition, the full Lagrangian approach of MEL method, which expresses nodes that follow the motion of water particles on the free surface with velocity $\vec{v}$ and $\frac{\delta}{\delta t} = \frac{\partial}{\partial t} + \vec{v}\bullet\nabla$, was applied (Eqs. (5) and (6)).

$$\frac{\delta\eta}{\delta t} = \frac{\partial\phi}{\partial z}, \tag{5}$$

$$\frac{\delta\phi}{\delta t} = -g\eta + \frac{1}{2}\left|\nabla\phi\right|^2 \tag{6}$$

where g is gravitational acceleration, ρ is water density. An artificial damping zone was applied on the free surface near the end wall of fluid domain to prevent reflected waves due to the rigid end wall. The wave particle velocity profile was used as the incident wave boundary condition.

$$\frac{\partial\phi}{\partial n} = \sum_{j=1}^{N}\left(-\frac{gA_jk_j}{\omega_j}\frac{\cosh[k_j(z+h)]}{\cosh k_jh}\cos(k_jx - \omega_jt + \varepsilon_j)\right) \tag{7}$$

where A, ω, k, h and ε denote wave amplitude, wave frequency, wave number, water depth, and random phase, respectively. Also, the impermeable boundary condition is applied to bottom and end wall.

3 Results and Discussion

Figure 1 compares the time series of traveling bi-chromatic wave elevations at two measurement points. Wave periods are set to T_1=1.9s, T_2=2.1s, and wave amplitude is A_1=A_2=0.08m. In the linear calculation, group waveforms generated from bi-chromatic waves were maintained and propagated. However, in the fully nonlinear calculation, the wave energy was concentrated within the group wave envelope as the generated group wave progresses. In addition, the difference between the phase angles of the linear result and the nonlinear result gradually increases as the wave propagates. The energy concentration and phase angle changes within the wave envelop, which can be seen in the nonlinear calculation results, were in good agreement with the previous experimental results [2].

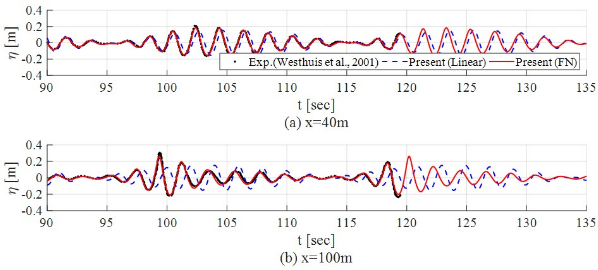

Figure 1: Comparison of time series of bi-chromatic waves (T_1=1.9s and T_2=2.1s)

Figure 2 compares the time series of irregular wave propagation measured at three measurement points in linear and fully nonlinear calculations. To generated irregular waves, JONSWAP spectrum was applied to incident wave boundary and a random phase was set to a range between 0 to 2π.

$$S(\omega) = \frac{5}{16}\frac{\omega_p^4 H_s^2}{\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^{\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]} \tag{8}$$

where the peak frequency was set to ω_p= 3.14rad/s, the significant wave height was set to H_s=0.1336m, the peak enhancement factor was set to γ=3.3. σ=0.07 for $\omega \le \omega_p$, and σ=0.09 for $\omega > \omega_p$, respectively. In the linear calculation, the shape of wave propagation was similar to the fully nonlinear result in the vicinity of the incident boundary (Fig. 2(a)). However, as the waves propagate, the linear results continue to maintain the initially formed waveform, but in the fully nonlinear calculation, wave energy was concentrated at a specific point similar to Figure 1, and the difference of phase angle between the two calculation results gradually increases.

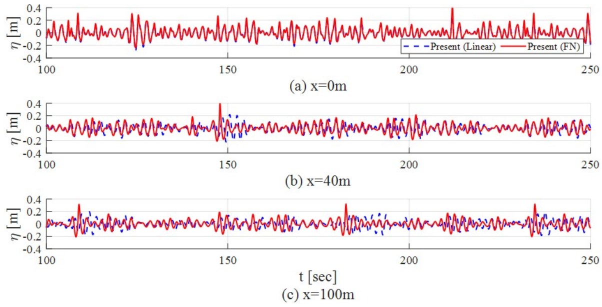

Figure 2: Time series of irregular waves at different measurement points

4 Conclusions

The spatial evolution of wave groups by wave–wave interactions was simulated numerically using the 2D-FN-NWT technique. The fully nonlinear calculation results were in good agreement with the experimental data on the phase angle change and wave energy concentration within the wave envelope. The spatial evolution of irregular waves was numerically analyzed using the verified numerical model. As the wave propagated, the linear results kept the initially formed wave form, but in the fully nonlinear calculation, the wave energy was concentrated at a specific point and the phase angle difference between the two calculation results gradually increased.

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