Boundary Integral Equation Methods for Optical Cloaking Models

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1 Abstract

Optical cloaking refers to making an object invisible by preventing the light scattering in some directions as it hits the object. There is interest in cloaking devices in radar and other applications. Developing a model to accurately capture cloaking comes with numerical challenges, however. We must determine how light propagates through a medium composed by multiple, thin layers of materials with different electromagnetic properties. In this paper we consider a multi-layered scalar transmission problem in 2D and use boundary integral equation methods to compute the field. The Kress product quadrature rule [2] is used to approximate singular integrals evaluated on boundaries, the Boundary Regularized Integral Equation Formulation (BRIEF) method [1] with Periodic Trapezoid Rule (PTR) is employed to treat nearly singular ones (off boundaries) appearing in the representation formula. Numerical results illustrate the efficiency of this approach, which may be applied to N arbitrary smooth layers.

Keywords: boundary integral methods, close evaluation, multi-layered media

2 Problem Setting

The light scattering by a plane wave $u^{in} = e^{ik_0\vec{\alpha}\cdot\vec{x}}$, with wavenumber $k_0 > 0$ and angle $\alpha \in [0, 2\pi]$ on N concentric smooth layers ($N \in \mathbb{N}^*$) in two dimensions can be written as a multi-layered scalar transmission problem: $\Delta u_j + k_j^2 u_j = 0$ in $\mathring{L}_j := L_j \backslash \{\Gamma_{j-1} \cup \Gamma_j\}$, $u_{j+1} = u_j$ on Γ_j, $\partial_{n_j} u_{j+1} = \frac{\varepsilon_{j+1}}{\varepsilon_j}\partial_{n_j} u_j$ on Γ_j, where u_j is the total field solution in the jth layer L_j and n_j is the outward unit normal on the boundary Γ_j (see Fig.1). Each layer is characterized by a permittivity ε_j, and wavenumber k_j. Note that the Sommerfeld radiation condition needs to be satisfied at infinity (in L_0).

3 Boundary Integral Equation System

Using the representation formula [1] and the transmission conditions, we represent the solution in layers L_j, $j \in [\![1, N-1]\!]$ as:

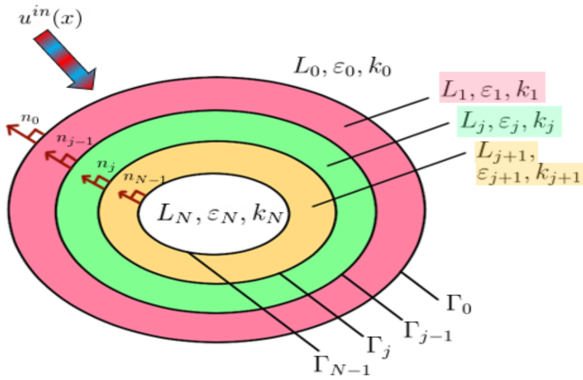

Figure 1: Sketch and notations for the problem.

$$u_j = - D_{j-1,j}[u_{j-1}] + \frac{\varepsilon_j}{\varepsilon_{j-1}} S_{j-1,j}[\partial_{n_{j-1}} u_{j-1}] + D_{j,j}[u_j] - S_{j,j}[\partial_{n_j} u_j] \quad \text{in } \mathring{L}_j, \tag{1}$$

Above, $D_{i,j}$, $S_{i,j}$ represent the double-layer potential and single-layer potential, respectively, defined on Γ_i for $i = j-1, j$, evaluated in L_j:

$$D_{i,j}[u_j](x) = \int_{\Gamma_i} \frac{\partial \Phi_j}{\partial n_i}(x,y) u_j(y) d\sigma_y, \; x \in \mathring{L}_j$$

$$S_{i,j}[\partial_{n_i} u_j](x) = \int_{\Gamma_i} \Phi_j(x,y) \partial_{n_i} u_j(y) d\sigma_y, \; x \in \mathring{L}_j$$

with the fundamental solution to the Helmholtz equation $\Phi_j(x,y) := \frac{i}{4} H_0^{(1)}(k_j|x-y|)$ with H_0^1 denoting the Hankel function of first kind. We also write $u_0 = u^{in} + D_{00}[u_0] - S_{00}[\partial_{n_0} u_0]$ and $u_N = -D_{N-1,N}[u_{N-1}] + \frac{\varepsilon_N}{\varepsilon_{N-1}} S_{N-1,N}[\partial_{n_{N-1}} u_{N-1}]$. As long as one knows the traces and normal traces $(u_j(y), \partial_{n_j} u_j(y))$ for $y \in \Gamma_j$, then one can evaluate the solution of the problem everywhere. To that aim, we solve the boundary integral equation (BIE) system shown below (obtained by evaluating the above representations on the boundaries and using known layer potential properties) [2].

4 Near-Boundary Evaluation with BRIEF

To treat singularities in the BIE system, we use the Kress product quadrature rule [2]. Using the BRIEF method with the PTR, we also treat

$$\begin{pmatrix} \frac{I}{2} - D_{00} & S_{00} & & 0 & 0 & 0 & 0 & \cdots \\ \frac{I}{2} + D_{01} & -\frac{\varepsilon_1}{\varepsilon_0} S_{01} & \ddots & -D_{11} & S_{11} & 0 & 0 & \cdots \\ D_{j-1,j} & -\frac{\varepsilon_j}{\varepsilon_{j-1}} S_{j-1,j} & & \frac{I}{2} - D_{j,j} & S_{j,j} & 0 & 0 & \cdots \\ 0 & 0 & & \frac{I}{2} + D_{j,j+1} & -\frac{\varepsilon_{j+1}}{\varepsilon_j} S_{j,j+1} & -D_{j+1,j+1} & S_{j+1,j+1} & \\ \vdots & \vdots & & & & \ddots & & \\ 0 & 0 & \cdots & D_{N-2,N-1} & -\frac{\varepsilon_{N-1}}{\varepsilon_{N-2}} S_{N-2,N-1} & \frac{I}{2} - D_{N-1,N-1} & S_{N-1,N-1} & \\ 0 & 0 & \cdots & 0 & 0 & \frac{I}{2} + D_{N-1,N} & -\frac{\varepsilon_N}{\varepsilon_{N-1}} S_{N-1,N} & \end{pmatrix} \begin{bmatrix} u_0 \\ \partial_{n_0} u_0 \\ \vdots \\ u_j \\ \partial_{n_j} u_j \\ \vdots \\ u_{N-1} \\ \partial_{n_{N-1}} u_{N-1} \end{bmatrix} = \begin{bmatrix} u^{in} \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

the nearly singular integrals in (1) close to the boundaries. To use BRIEF [1], we consider auxiliary functions ψ_j, $j \in [\![0, N-1]\!]$:

$$\psi_j(x) = u_j(x_i^b) g_j(x) + \partial_{n_i} u_j(x_i^b) f_j(x), \quad x \in L_j,$$

with g_j and f_j satisfying the associated Helmholtz equation in L_j with the boundary conditions $g_j(x_i^b) = 1$, $\partial_{n_i} g_j(x_i^b) = 0$, $f_j(x_i^b) = 0$, $\partial_{n_i} f_j(x_i^b) = 1$, with $x_i^b \in \Gamma_i$, the point on the boundary closest to x ($x = x_i^b + n_i \ell$, with $\ell \neq 0$). Using Green's identities, we subtract the nearly singular behaviors using ψ_j as follows:

$$\begin{aligned} u_j &= \psi_j - D_{j-1,j}\left[u_{j-1} - \psi_{j-1}\right] \\ &+ \frac{\varepsilon_j}{\varepsilon_{j-1}} S_{j-1,j}\left[\partial_{n_{j-1}} u_{j-1} - \partial_{n_{j-1}} \psi_{j-1}\right] \\ &+ D_{j,j}\left[u_j - \psi_j\right] - S_{j,j}\left[\partial_{n_j} u_j - \partial_{n_j} \psi_j\right]. \end{aligned}$$

As illustrated in Figures 2-3 the error (or the field's amplitude) is reduced at the vicinity of the boundaries with BRIEF. The incident field is a plane wave with angle $\alpha = \frac{\pi}{4}$ and for the discretization a body-fitted grid with 100 points on each boundary and 100 points along the radial direction was used.

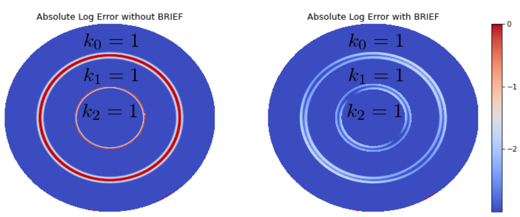

Figure 2: Absolute log error for standard solution (left) and BRIEF solution (right) for homogenous case, where boundaries are circles of radii 2 and 1.

5 Ongoing Work

Future goals include reducing the time needed to generate multi-layered models, which will be done with parallel computing; investigating relevant applications with effects on the solvability

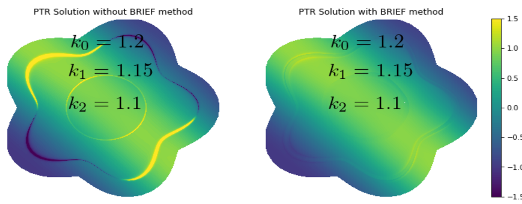

Figure 3: Standard solution (left) and BRIEF solution (right) for nonhomogeneous case, where boundaries are a circle of radius 1 and a star parameterized by $y(t) = (2 + 0.3\cos(5t))\,\langle \cos(t), \sin(t) \rangle$, $t \in [0, 2\pi]$.

of the BIE system. In particular, in order to efficiently simulate optical cloaking, we will consider different electromagnetic properties [3] and different (non convex) boundary shapes. One can also create lossy cloaking devices. In that case, one choose some $k_j \in \mathbb{C}$ with $\Im(k_j) > 0$, and other quadratures than Kress will be needed to preserve accuracy. Extensions to 3D will also be considered.

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