Modeling and simulation of nonlinear wave propagation in ultrasound imaging

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Abstract

Nonlinear ultrasound imaging has become increasingly important in medical but also industrial areas, as it makes the ultrasound beam analysis more accurate and enhances the quality of images. We develop a three dimensional model for nonlinear ultrasound imaging and a numerical algorithm to simulate pressure waveforms based on the KZK-equation. In our case we model the propagation of three dimensional sound beams in water for a rectangular source. With an eigensystem approach for the Laplacian operator on a rectangular domain and an operator splitting method the effects of nonlinearity and diffraction can efficiently be taken into account. To demonstrate the accuracy of the developed model it has been implemented in MATLAB and simulated results are compared to acoustic output measurements performed by a hydrophone.

Keywords: nonlinear wave propagation, ultrasound imaging, KZK-equation

1 Introduction

The pressure waves of ultrasound beams have strong nonlinearities at higher intensities. Figure 1 shows a beam sent out by a linear probe, the intensity of the pressure and the corresponding pressure waveforms at different depths.

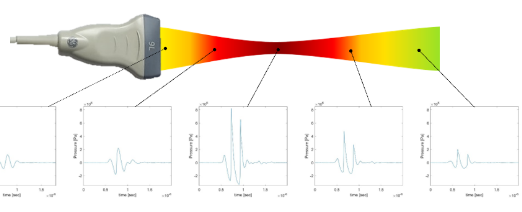

Figure 1: Acoustic measurement: overview

The wave steepens, the amplitude of the compressional peak gets higher, whereas the rarefactional peak gets shallower. The waveform distorts because of increasing energy in higher harmonic frequencies. To model this behavior, we consider the KZK-equation as it accounts for diffraction, nonlinearity and absorption and uses a paraxial approach which is especially suitable for directive sound beams [2]. We investigate the wave propagation in water, so attenuation can be neglected and the KZK-equation in integrated form in terms of the pressure p reduces to

$$\partial_z p = \frac{c_0}{2}\int_{-\infty}^{\tau} \Delta_\perp p \, \mathrm{d}\tilde{\tau} + \frac{\beta}{2\rho_0 c_0^3}\partial_\tau p^2, \qquad (1)$$

where τ is the retarded time, c_0 the speed of sound, ρ_0 the density and β the nonlinearity parameter. The direction of propagation is z, so x, y are orthogonal coordinates and $\Delta_\perp = \partial_x^2 + \partial_y^2$. An efficient numerical method to solve this equation is an operator splitting approach [5], where each physical effect, i.e.

$$\partial_z p = \frac{c_0}{2}\int_{-\infty}^{\tau} \Delta_\perp p \, \mathrm{d}\tilde{\tau}\,, \qquad (2)$$

$$\partial_z p = \frac{\beta}{2\rho_0 c_0^3}\partial_\tau p^2, \qquad (3)$$

is assumed to operate independently of the other over sufficiently small propagation distances.

2 Model equation

The shape of the beam strongly depends on the geometry of the probe [4] which we assume to be rectangular. Since it is symmetrical around its central axis we only consider the first quadrant. Looking at the classical eigenvalue problem for the Laplace operator with Neumann boundary conditions [1] on $\Omega = [0, a] \times [0, b]$, the eigenvalues and eigenvectors are explicitly given by

$$\lambda^{jk} = \frac{\pi^2 j^2}{a^2} + \frac{\pi^2 k^2}{b^2},$$

$$\phi^{jk}(x, y) = \frac{2}{\sqrt{ab}} \cos\left(j\pi\frac{x}{a}\right) \cos\left(k\pi\frac{y}{b}\right).$$

The overall pressure at any time τ and position x, y, z is given by

$$p(x, y, z, \tau) = \sum_j \sum_k p^{jk}(z, \tau)\phi^{jk}(x, y). \qquad (4)$$

With these relations solving (2) is equivalently to solve for all m, n the ODE

$$p^{mn}(z, \tau) = -\frac{2}{c_0\lambda^{mn}}\, p^{mn}_{z\tau}(z, \tau). \qquad (5)$$

From (4) one can also derive an expression for the source condition.

3 Numerical results

We start our algorithm by applying a discrete cosine transform to the initial data, compare (4). Then we perform the diffraction step over a sufficiently small distance of propagation Δz, where (5) has been discretized with an implicit finite difference scheme. Next, we change back into time domain by applying the inverse cosine transform. Then we take the effect of nonlinearity with a discretized version of (3) over Δz into account and one step is completed. Now, this algorithm has to be carried out until the final plane at depth z_{out} is reached.

The following simulations were performed with MATLAB based on the proposed algorithm where the initial data was calculated by FIELD II [3]. Figure 2 shows numerical simulations of pressure waveforms on the central axis with the corresponding frequency spectra at various distances based on the proposed numerical algorithm.

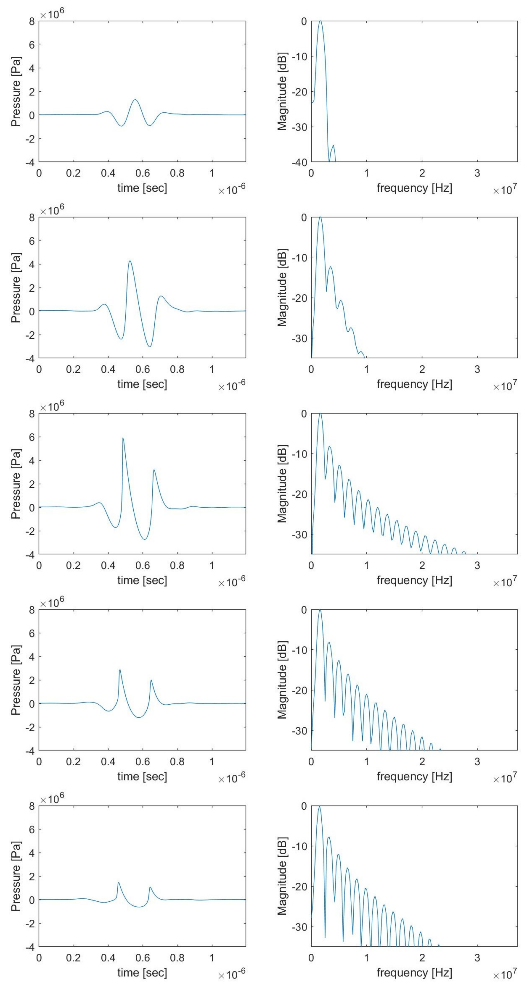

Figure 2: On-axis pressure waveforms with corresponding frequency spectra at different depths

Finally, in the following figures we compare on-axis pressure waveforms from acoustic measurements (blue) and simulations (orange) at different depths.

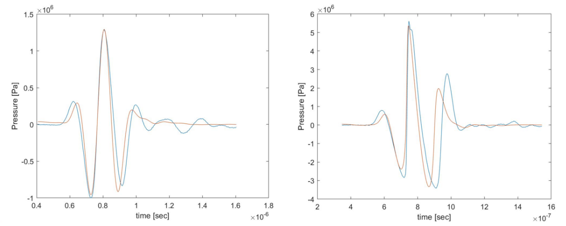

Figure 3: waveform at z=1.5mm, z=16.5mm

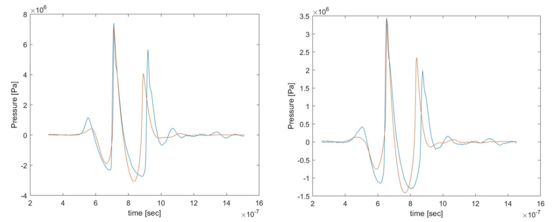

Figure 4: waveform at z=26.5mm, z=39.5mm

4 Outlook

Future work is the derivation of a paraxial model based on the KZK-equation for vibro-acoustic imaging, do simulations and to put it into the mathematical framework of inverse problems (more precisely, coefficient identification in PDEs) and regularization.

References

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