Homogenisation of perforated plates

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Abstract

This paper considers the effects of periodic apertures perforating a thin plate and the appropriate homogenised boundary condition one should apply when such a plate is placed in flow. Specific attention is placed on a boundary condition applicable to acoustic scattering by an incident plane wave. The impact of homogenisation on the far-field scattered noise is investigated using the Wiener-Hopf technique. An explicit modelling of a perforated plate via Mathieu function collocation is also introduced.

Keywords: Rayleigh conductivity, impedance, homogenisation

1 Introduction

Aerofoil leading-edge noise may be attenuated by introducing permeability through apertures which perforate the entire thickness of the aerofoil. In particular, we consider the effect of periodically placed apertures, which have been seen experimentally to reduce noise.

The boundary condition upon such a plate is studied by replacing it with an effective impedance boundary condition, leading to a homogeneous model for the entire plate. We focus under the assumption that the size of each perforation is small compared to the distance between apertures, which is in turn much smaller than the wavelength of the incoming acoustic waves. The relevant parameter in the present model will be the Rayleigh conductivity: a measure of compliance, it characterises the fluctuating volume flow rate through the aperture [2]. In this paper, we shall investigate the far-field effects of different homogenised boundary condition models and how they compare against the computationally expensive route of modelling a perforated plate explicitly.

2 Body of the paper

Howe [2] originally assumes continuous displacement across the plate, so in the linearised approximation, the z-component of displacement of particles $\zeta(x,y)^{-i\omega t}$ lying just above and below $z=0$ vanish outside the circular aperture and equal the displacement of the vortex sheet within the aperture. His method leads to an integro-differential equation for the non-dimensionalised displacement Z;

$$\int_S \frac{Z(\xi',\eta')\mathrm{d}\eta'\mathrm{d}\xi'}{\sqrt{(\xi-\xi')^2+(\eta-\eta')^2}} = 1+\alpha_1(\eta)e^{i\sigma_1\xi} + \alpha_2(\eta)e^{i\sigma_2\xi}, \quad (1)$$

where S is the unit circle, (ξ,η) are the non-dimensionalised physical coordinates, and σ_i are the Kelvin-Helmholtz wavenumbers of instability waves on the vortex sheet. The amplitudes $\alpha_i(\eta)$ are set by the Kutta condition at the leading edge. The solution of (1) may then be used to determine the Rayleigh conductivity via

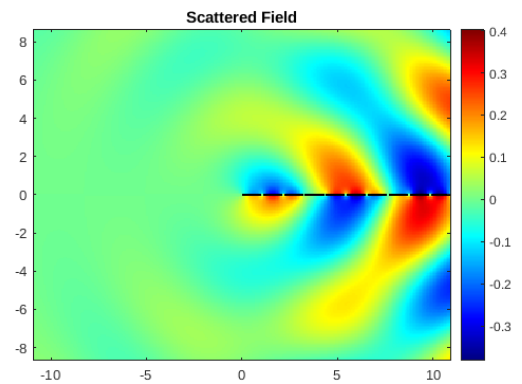
$$K_R = \pi R \int_S Z(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta. \quad (2)$$

Conversely to Howe's assumption of continuous displacement, one can instead consider continuity of particle velocity [3, 7]. This route follows the same derivation, but merely includes an alternative jump condition between the two sides of the plate. The model considers a plate of thickness T and an aperture with characteristic length B. The integro-differential equation to solve following this model is for the non-dimensional *normal velocity* Z;

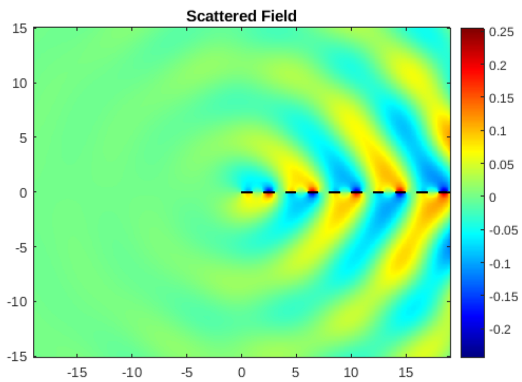
$$\int_{S_0} \frac{Z(X',Y')\mathrm{d}X'\mathrm{d}Y'}{\sqrt{(X-X')^2+(Y-Y')^2}} - 2i\pi\mathrm{Sr}\frac{T}{B}\int_{LE}^{X} Z(X',Y)e^{2i\mathrm{Sr}(X-X')}\mathrm{d}X' = 1+\alpha(Y)e^{2i\mathrm{Sr}X}. \quad (3)$$

S_0 is the non-dimensionalised version of the orifice with non-dimensional coordinates (X,Y), LE denotes the leading edge of the orifice, Sr is the Strouhal number. Again, $\alpha(Y)$ is set by imposing the Kutta condition at the leading edge.

We discuss the effectiveness of these homogenised models by comparing them with an explicit model. The Helmholtz equation in elliptic coordinates is separable, admitting a series solution expanded in terms Mathieu functions. Colbrook [4] developed a model for scattering by multiple plates. For our purposes, we construct a single perforated plate by aligning multiple, rigid, plates colinearly to create a long perforated plate. The result is a scattered profile of a plane wave by a one-dimensional perforated plate. The following figures show the field for differing sizes of the perforations ϵ, for the same incident wave.



(a) $\epsilon = 0.1$



(b) $\epsilon = 1$

These half-plane problems are then compared against the homogenised impedance parameters through solving the Wiener-Hopf problem for a general compliant plate, choosing the compliance parameter as dictated by each model. The resulting kernels from the problem without mean flow [1] and with mean flow [8] are factorised numerically. The inverse Fourier transforms are treated with the method of steepest descent to observe the far-field directivities.

3 Future work

Future work will take into the account boundary layers induced by the mean flow, which have been so far neglected by existing impedance models, or simplified to a vortex sheet. We follow the method of Brambley using the method of matched asymptotic expansions on the equations of motion for a perfect gas, in Cartesian coordinates. This will produce an effective impedance parameter from which we will be able to model the flow as uniform; the information pertaining to the boundary layer will be stored in the effective impedance.

4 References

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