Spectral Difference method On Structured-Grids for Maxwell's Equations in Time Domain.

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Abstract

This article introduces a new way to discretize Maxwell's equations. It is a discontinuous high-order method based on local polynomial interpolations, named the Spectral Difference method. This approach mainly differs from the standard Discontinuous Galerkin method by solving the strong form of the equation, instead of the weak form. This article gives the main lines of the Spectral Difference method for a 1D conservation law and explains how it applies to transient Maxwell's equations. The method is then evaluated on a test-case with well-known analytical solution.

Keywords: High-order method, Spectral Difference, Maxwell's equations, Time domain.

1 Introduction

The Spectral Difference (SD) method is a discontinuous high-order method based on local polynomial interpolations. This method was introduced in [1] as a scheme that is conservative, high-order, geometrically flexible, computationally efficient and simply formulated. It solves the strong form of the equation, as in Finite Difference. This method has been widely explored in Computational Fluid Dynamic (CFD) as an alternative to the Discontinuous Galerkin method (DG) [2]. The present article starts from the formulation in [1] for Maxwell's equations but accounts for specific location of the degrees of freedom associated with stable SD schemes [3]. This paper is organized as follows. Section 1 explains how the SD method works for a 1D conservation law and how it generalizes to Maxwell's equations. Section 2 shows a test-case, on a non-Cartesian grid, with well-known analytical solution.

2 Spectral Difference method

In 1D, the main idea of the SD method, for a conservative system ($\partial_t u = -\partial_x F(u)$), is to compute the conserved variables u as a polynomial of degree p ($p \in \mathbb{N}^*$) and to compute the flux as a polynomial of degree $p+1$ so that solution and flux divergence are both polynomials of degree p. Solution and flux polynomials are defined by Lagrange interpolation using two sets of points, $p+1$ *solution points* and $p+2$ *flux points* (Figure 1).

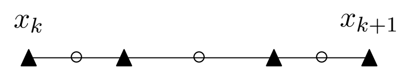

Figure 1: Example of point positions over a segment, for $p = 2$: *solution points* (○) and *flux points* (▲).

The stability of the method only depends on the location of the latter [3]. In 1D, two of the *flux points* must be taken as the segment end points, the p remaining are taken inside the element. The formulation is shown linearly stable for any degree p [4]. The SD algorithm consists of four steps:

1. The solution is extrapolated to the *flux points*.
2. For the two boundary points, the flux is the solution of a Riemann problem. On the internal points, the flux is a linear combination of the unknowns.
3. The flux polynomial is interpolated from the *flux points*.
4. The flux is differentiated at the *solution points* in order to update the solutions.

For 2D/3D configurations, problems are considered using tensorized 1D formulation. An efficient implementation can easily be obtained using matrix/vector products.

The SD formulation is applied to Maxwell's equations written in conservative form; they consist of two coupled linear advection equations (in 1D, $u = (E, H)$ and $F(u) = (H, E)$). From there, everything described above applies: each component of the fields $\mathbf{E}$ and $\mathbf{H}$ is approximated by a polynomial of degree p, and the interpolations are taken in every directions using a tensorial rule.

3 Numerical results

As a primary validation, the method is tested by considering a propagating mode (here the $(1, 1, 0)$ mode) inside the unit cube of $\mathbb{R}^3$. For computation, $p = 3$. The mesh is a structured non-Cartesian grid (Figure 2), with 32,768 degrees of freedom and 512 cells.

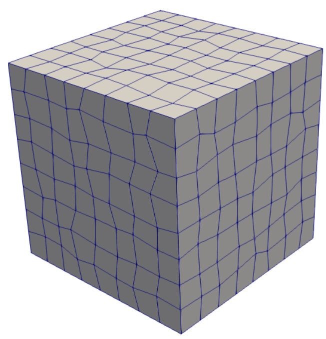

Figure 2: Structured non-Cartesian mesh used for the simulation. 32,768 degrees of freedom, 512 cells.

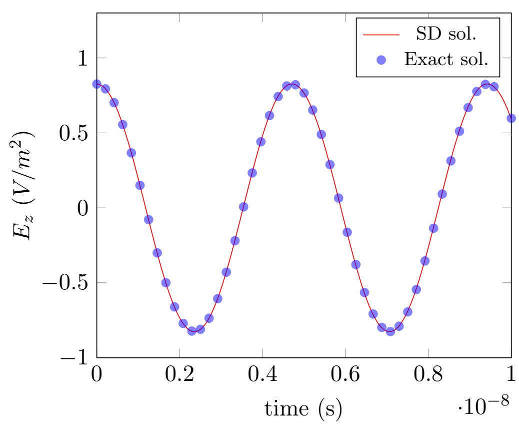

Figure 3: Comparison of the numerical solution given by the SD method for $p = 3$ (red curve), and the analytical solution (blue dots) for the mode $(1, 1, 0)$.

Figure 3 shows the good agreement between the SD numerical solution and the analytical solution. Other configurations will be presented during the conference.

4 Conclusion

In this article, a new way (the SD method) to discretize Maxwell's equations is introduced and validated for a cavity mode by using a non-Cartesian grid. For the presentation further details will be given on the formulation of the SD method, in particular stability results and comparisons with both FDTD [5] and DG schemes [6].

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