# Waveguide acoustic black holes: non-helpful and helpful damping mechanisms

**Martin Berggren**[1,*], **Abbas Mousavi** [1], **Eddie Wadbro**[2]
[1]Department of Computing Science, Umeå University, Umeå, Sweden
[2]Department of Mathematics and Computer Science, Karlstad University, Karlstad, Sweden
*Email: martin.berggren@cs.umu.se

### Abstract

Motivated by puzzling experimental results for a waveguide acoustic black hole, we investigate through computations which sources of damping that are significant for a successful function of the device. As opposed to analogous structural devices, the addition of damping material towards the end of the device has essentially no effect. Our results suggest instead that damping at the outer tube in combination with viscothermal boundary-layer losses are the most critical sources of damping.

*Keywords:* waveguide, acoustic black hole

## 1 Introduction

Mironov [3] introduced a broad-band damping device for a beam or plate, usually referred to as an *acoustic black hole*, which very efficiently damps vibrations above a given critical frequency. A gradual tapering of the structural thickness causes a decrease in wave velocity and an increase in amplitude, which means that a small amount of damping material at the thin side can effectively dampen the vibrations.

An analogous device for guided wave propagation in air has been suggested and analyzed by Mironov and Pislyakov [4]. The experimental investigations by El Ouahabi et al. [5] of this type of device gave quite surprising results. The performance, in terms of the reflection coefficient at the inlet, agrees reasonably well with predictions [4] even without explicitly added damping material. However, unlike the structural counterpart, the addition of damping material at the device's peripheral end does *not* significantly improve performance (figure 1).

Here we systematically investigate, through numerical simulations, the relative importance of possible sources of damping in this device, and we suggest likely candidates for the most important damping mechanisms. All numerical investigations are carried out by solving the Helmholtz equation for the acoustic pressure in 2D axial symmetry for the geometry illustrated in figure 2. A simple radiation condition at $\Gamma^L$ imposes a right-traveling planar wave and absorbs reflected planar waves. We restrict the analysis to the frequency range below the first higher (circumferential) mode at 866 Hz.

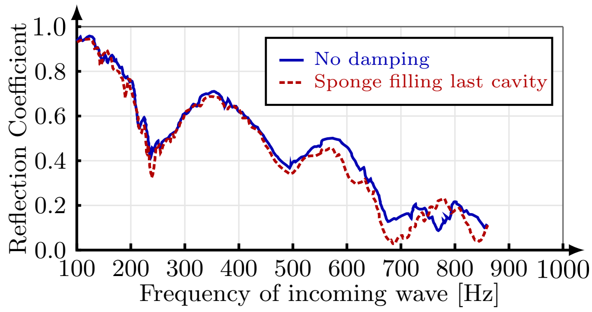


Figure 1: Measured reflection coefficients (data from El Ouahabi et al. [5], used with permission)

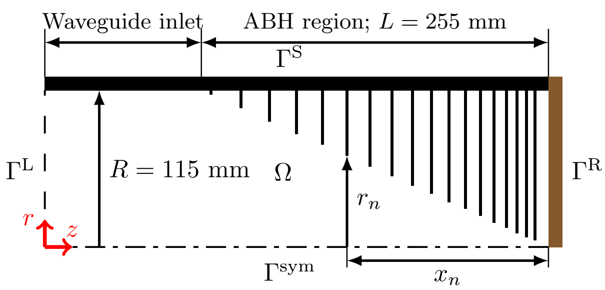


Figure 2: The axisymmetric acoustic black hole

## 2 Non-helpful damping

The idea behind the design of figure 2 is to slow the propagation speed and increase the wave amplitude as the wave progresses towards the right end. Theory predicts an operational range for frequencies $f \geq f_{\text{crit}} = c\pi/L \sim 425$ Hz [2], where $c$ is the speed of sound. In analogy to structural black holes, it seems reasonable to focus on damping effects at the end of the device. We therefore consider a surface-impedance boundary condition at $\Gamma^R$, a wooden wall in the experiments, and simulate damping materials in the last cavity through a Rayleigh-damping model. As can be seen in figure 3, we note a complete failure of this setup to provide any damping

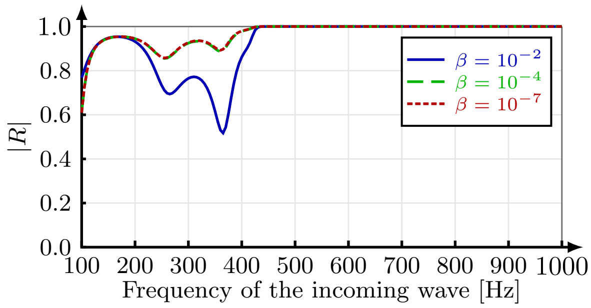


Figure 3: Reflection coefficient with wall damping at $\Gamma^R$ and damping material in the last cavity; $\beta$ is a parameter in the Rayleigh-damping model

in the predicted operational range. Even very aggressive damping in this region has no helpful effect above $f_{\text{crit}}$.

## 3 A somewhat helpful damping effect

Due to the radial rings, the device contains a quite large area of sound-hard material. It is therefore reasonable to expect losses due to visco–thermal boundary-layer effects. To account for these, we use the model devised by Berggren et al. [1], which, in cylindrical symmetry, models these effects through the boundary condition

$$-\delta_V \frac{i-1}{2} \nabla_T \cdot (r \nabla_T p) + \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} rp + r\frac{\partial p}{\partial n} = 0, \tag{1}$$

where $\gamma$ is the heat capacity ratio, $k$ the wavenumber, $\delta_V$ and $\delta_T$ the wave-number-dependent viscous and thermal boundary-layer thicknesses, and $\nabla_T$ the tangential gradient operator. Including also these losses, in addition to the previous, we indeed obtain a discernible effect on the reflections, as is clear from figure 4.

## 4 A crucially helpful damping effect

As designed by Mironov and Pislyakov [4], the purpose of the concentric rings in figure 2 is to provide an increasingly compliant wall admittance. Indeed, this property holds reasonably well for low frequencies. However, the cavities are deep enough to cause a more complicated wall-admittance load at higher frequencies. A combined compliant/inertial effect creates local resonances in the cavities, located progressively further from the far end the higher the frequency, explaining why damping material towards the end has no effect. The high localized sound pressures in the ring cavities likely generate losses

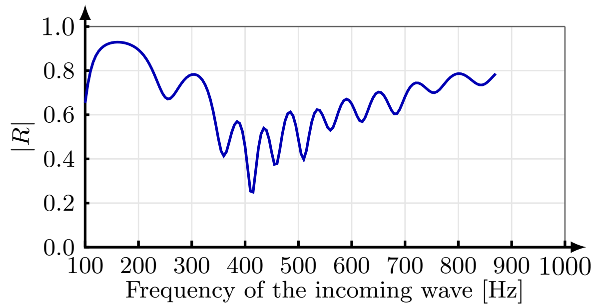


Figure 4: Reflection coefficient when also taking visco-thermal boundary-layer losses into account

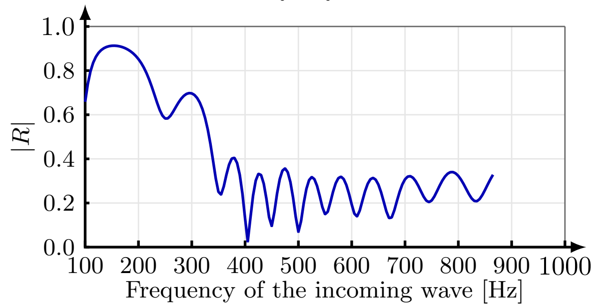


Figure 5: Reflection coefficient when also assuming a small amount of surface damping at the outer tube

at the outer (plastic) tube, a conjecture supported by the reflection spectrum in figure 5, where a small amount of surface damping also has been added to the outer tube. Although we lack data to make a quantitatively accurate prediction, our results supports the hypothesis that the main sources of damping that makes the waveguide acoustic black hole work is a combination of losses at the outer tube and visco–thermal boundary-layer losses. A natural next step would be to apply numerical optimization to the geometry and the location of supplemental damping to further improve performance.

## References

[1] M. Berggren, A. Bernland, D. Noreland, *J. Comput. Phys.* 371: 633–650, 2018.

[2] Y. Mi, W. Zhai, L. Cheng, C. Xi, X. Yu, *Appl. Phys. Lett.* 118, p. 114101, 2021 .

[3] M. Mironov, P *Sov. Phys. Acoust.* 34:318–319, 1988.

[4] M. A. Mironov and V. V. Pislyakov, *Acoust. Phys.*, 48:347–352, 2002.

[5] A. El Ouahabi, V. Krylov, D. O'Boy, Experimental investigation of the acoustic black hole for sound absorption in air, Proc. 22nd Int. Congr. Sound Vibrat., Florence, Italy, 2015.