Iterative helioseismic holography-Inversions for solar differential rotation

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Abstract

Helioseismic inversions are still challenging due to the high level of noise based on the stochastic nature of solar oscillations and the large amount of input data. In order to deal with the immense size of data one usually is in need of some a priori averaging. Traditional helioseismic holography averages the data by backpropagating the surface data to a target location in the solar interior. Therefore holography provides feature maps, but no quantitative reconstructions. We extend helioseismic holography to a full converging regularization method by linking the physically motivated backpropagation operator with the adjoint of the Fréchet derivative of an appropriate operator mapping to the cross-covariance operator.

Keywords: helioseismology, holography, inverse problems

1 Introduction

In helioseismology one analyzes solar oscillations at the surface (e.g. line-of-sight velocities) in order to learn about subsurface properties. Data of the inverse problems are cross-correlations of the observed wavefield ψ between two points $\boldsymbol{r_1}, \boldsymbol{r_2}$ on the solar surface caused by stochastic turbulent convection.

$$C(\boldsymbol{r_1}, \boldsymbol{r_2}, \omega) = \frac{1}{N} \sum_{i=1}^{N} \psi_i(\boldsymbol{r_1}, \omega)\psi_i(\boldsymbol{r_2}, \omega)^*,$$

where ω denotes the frequency. The cross-correlation is a five-dimensional data set of immense size, which is unfeasible to store and invert directly. Therefore, one is generally in need of a priori averaging in space and frequency. Traditional approaches like time distance helioseismology achieve this goal by reducing the cross-correlation to a small number of physically interpretable quantities (for example travel times) with an acceptable signal-to-noise ratio. In contrast to time distance helioseismology, helioseismic holography first propagates the wavefield back to a target location and afterwards locally cross-correlate the backpropagated wave field in order to image the quantity of interest. This way the whole cross-correlation data is used implicitly without computing the cross-correlation explicitly. Despite to the great success of helioseismic holography, it is no quantitative regularization method (see figure 1).

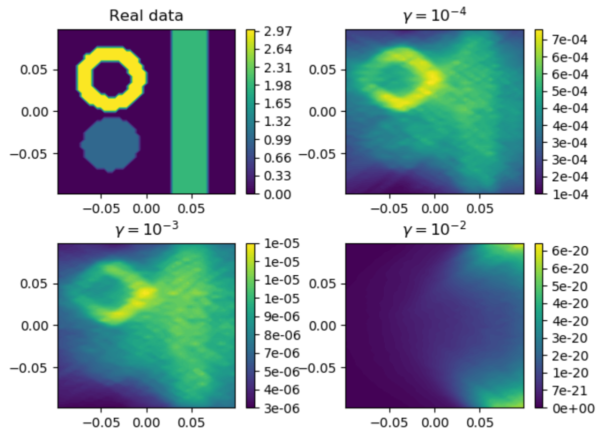

Figure 1: It is shown Lindsey-Braun holography for a uniform medium with wavenumber $k = 100 + i\gamma$. We have chosen 100 receivers uniformly spaced at a circular surface of radius 1. Although the main features are visible for small wave attenuation, the holographic image is wrong on scales.

2 Helioseismic holography

The wavefield can be described by a Helmholtz equation [1]:

$$-(\Delta + k^2)\psi - \frac{2i\omega}{\rho^{1/2}c}\rho\boldsymbol{u} \cdot \nabla \left(\frac{\psi}{\rho^{1/2}c}\right) = s,$$

where $\boldsymbol{u}$ is the flow field and s the stochastic source term. The density ρ, the sound speed c and the wave attenuation γ are modeled by the Solar Model S, smoothly extended to the atmosphere [2]. The local wavenumber k takes the form

$$k^2 = \frac{\omega^2 + 2i\omega\gamma}{c^2} - \rho^{1/2}\Delta\rho^{-1/2}.$$

We use a radiation boundary condition assuming an exponential decay in near-surface layers. The holograms ϕ_α can be computed from the measured wavefield on the observable surface:

$$\phi_\alpha(\boldsymbol{x},\omega) = \int_A H_\alpha(\boldsymbol{x},\boldsymbol{y},\omega)\psi(\boldsymbol{y},\omega)d\boldsymbol{y},$$

where H_α is a wave propagator (usually defined in terms of the Green function), which backpropagates the wavefield on the surface to a target location in the solar interior. Due to the stochastic nature of solar oscillations and within the holograms we are interested in the hologram intensity:

$$I_{\alpha,\beta}(\boldsymbol{r},\omega) = \phi_\alpha(\boldsymbol{r})^*\phi_\beta(\boldsymbol{r})$$

Perturbations to the solar medium can be expressed in terms of sensitivity Kernels $\mathcal{K}_{\alpha,\beta}$:

$$\mathbf{E}\left[\delta I_{\alpha,\beta}(\boldsymbol{x},\omega)\right] = \int_\Omega \mathcal{K}_{\alpha,\beta}(\boldsymbol{x},\boldsymbol{y})\delta q(\boldsymbol{x})d\boldsymbol{x},$$

where $q \in \{\rho, c, \gamma, \boldsymbol{u}\}$. Usually the backpropagators are chosen, such that the sensitivity Kernels become localized.

3 Iterative holography

In contrast to the traditional approach of holography we will fix the wave propagators by the scalar wave equation and improve the holograhic images by iterations. The covariance of the Doppler velocities can be written in the form [3]:

$$C(q) := \mathbf{Cov}[\psi] = TL_q^{-1}\mathbf{Cov}[s](L_q^*)^{-1}T^*,$$

where T is a trace operator acting as projection to the surface and C the covariance operator. The Fréchet derivative is given by:

$$C'[q]\delta q = -2Re\left(H_\alpha^*[\partial_q L_q \delta q]H_\beta\right),$$
$$H_\alpha^* = TL_q^{-1}, \quad H_\beta^* = L_q^{-1}\mathbf{Cov}[s](L_q^*)^{-1}T^*$$

where $ReA = \frac{1}{2}(A + A^*)$. The adjoint with respect to the Frobenius norm takes the form

$$C'[q]^*\hat{C} = -2[\partial_q L_q]^*\left((H_\alpha(Re\hat{C})TL_q^{-1}H_\beta\right)$$

The operators H_α and H_β can be interpreted as backpropagator, whereas $(\partial_q L_q)^*$ takes the role of a local correlation operator. This setting links the adjoint of the Fréchet derivative of the covariance operator to the wave propagation of traditional seismic holography. In this way we can extend helioseismic holography to a converging iterative regularization method, where the first iteration coincides with the classical holographic image.

4 Differential rotation

Due to the large computational costs for solving the forward problem, caused by the strong gradients close to the solar atmosphere, one usually solves linearized problems in local helioseismology. Our approach allows an application to nonlinear problems like inversions for solar differential rotation or solar convection. Usually differential rotation is measured by the frequency splitting caused by a rotating medium. Although this inversion has led to exciting results as the detection of the tachocline and NSSL, there are still some questions open. In particular frequency splitting does not allow us to invert for the antisymmetric part of the differential rotation. We have achieved very encouraging numerical results for the differential rotation in the solar convection zone (see figure 2).

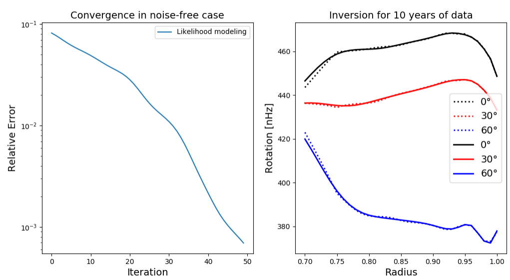

Figure 2: Preliminary inversion results for noise-free data (left panel) and simulated realistic data with noise level corresponding to 10 years of observation.

References

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