A layer potential approach to functional and clinical brain imaging

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Abstract

We study the inverse problems of source recovery and cortical mapping for sEEG, EEG and MEG under the quasi-static approximation of Maxwell's equations. These problems are known to be ill-posed due to the existence of silent source, *i.e*, those non-zero sources that generate null field. In addition the collected data are corrupted with noise that has to be corrected for, hence regularised Tikhonov problems will be solved. We make use of single and double layer potentials to express the electro-magnetic (EM) fields associated with a source which allows to simultaneously solve the source recovery and cortical mapping problems for a particular modality or coupled data. The expression of the electro-magnetic fields we use make the different couplings of sEEG, EEG and MEG data direct and complementary. Numerical results of experiments performed using meshes of realistic head geometries will be presented.

Keywords: ElectroEncephaloGraphy (EEG), stereo-ElectroEncephaloGraphy (sEEG), MagnetoEncephaloGraphy (MEG), inverse problems, brain imaging, cortical mapping

1 Introduction

We model the head as a layered conductor with different electric conductivities between layers which are constant in each layer. These homogeneous layers are the brain, skull and scalp, with the brain subdivided into the grey and white matter regions. Brain activity is modelled as vector-fields (sources) $\boldsymbol{\mu}_0 \in [\mathcal{M}(\Sigma_0)]^3$, $\mathcal{M}(\Sigma_0)$ being a Banach space whose elements are supported on grey/white matter interface, the surface Σ_0. Let Σ_i be the closed surfaces on which the electric conductivity is discontinuous, σ_i^- and σ_i^+ are the electric conductivities inside and outside Σ_i, respectively, and $\mathcal{H}_i$ is the Haussdorf measure on Σ_i then the electric potential associated with $\boldsymbol{\mu}_0$ at $x \in \mathbb{R}^3$ is:

$$\begin{aligned}\sigma(x)\phi(x) = \frac{1}{4\pi}\int \frac{(x-y)}{|x-y|^3}\cdot d\boldsymbol{\mu}_0(y) - \\ \sum_{i=1}^{n+1} \frac{\sigma_i^- - \sigma_i^+}{4\pi}\int_{\Sigma_i} \phi(y)\nu(y)\cdot\frac{(x-y)}{|x-y|^3}\, d\mathcal{H}_i(y).\end{aligned} \tag{1}$$

The magnetic flux density associated with $\boldsymbol{\mu}_0$ at $x \in \mathbb{R}^3$ is:

$$\begin{aligned}\mathbf{B}(x) = \frac{\mu}{4\pi}\int \frac{(x-y)}{|x-y|^3}\times d\boldsymbol{\mu}_0(y) - \\ \mu\sum_{i=1}^{n+1} \frac{\sigma_i^- - \sigma_i^+}{4\pi}\int_{\Sigma_i} \nu(y)\times\frac{(x-y)}{|x-y|^3}\phi(y)\, d\mathcal{H}_i(y),\end{aligned} \tag{2}$$

where the $\phi \in L^2(\Sigma_i)$ are as computed in (1).

2 Inverse Problems

We aim to solve problems of the following form with measurement in $D_1 \subset \mathbb{R}^3$ depending on the modalities:

Problem 1 *Given data $f \in L^2(D_1)$ and parameters $\lambda, \lambda_i > 0$, $i = 1, 2, \ldots, n+1$, find*

$$\begin{aligned}&(\boldsymbol{\mu}_0, \phi_1, \phi_2, \ldots, \phi_{n+1})_\lambda = \\ &\operatorname*{arg\,inf}_{(\boldsymbol{\mu}_0, \phi_1, \phi_2, \ldots, \phi_{n+1})\in S} \mathcal{T}_{f,\lambda}(\boldsymbol{\mu}_0, \phi_1, \phi_2, \ldots, \phi_{n+1}),\end{aligned}$$

where

$$\begin{aligned}\mathcal{T}_{f,\lambda} := &\left\|\boldsymbol{\mathcal{F}}_1(\boldsymbol{\mu}_0, \phi_1, \phi_2, \ldots, \phi_{n+1}) - f\right\|^2_{L^2(D_1)} \\ &+ \left\|\boldsymbol{\mathcal{F}}_2(\boldsymbol{\mu}_0, \phi_1, \phi_2, \ldots, \phi_{n+1})\right\|^2_{D_2} \\ &+ \lambda\left\|\boldsymbol{\mu}_0\right\|_{TV} + \sum_{i=1}^{n+1}\lambda_i\left\|\phi_i\right\|^2_{L^2(\Sigma_i)},\end{aligned}$$

where $\boldsymbol{\mathcal{F}}_1$ corresponds to the EM data, $\boldsymbol{\mathcal{F}}_2$ to the cortical mapping, D_2 is the Hilbert space

$$L^2(\Sigma_1) \times \cdots \times L^2(\Sigma_n) \times L^2(\Sigma_{n+1}) \times L^2(\Sigma_{n+1})$$

which corresponds to the electric potential on the interfaces Σ_i, $i = 1, 2, \ldots, n+1$ and S is the product Banach space

$$[\mathcal{M}(\Sigma_0)]^3 \times L^2(\Sigma_2) \times \cdots \times L^2(\Sigma_n) \times L^2(\Sigma_{n+1}).$$

We propose to use an alternating minimisation procedure generating a sequence of solutions

$$\left\{(\boldsymbol{\mu}_0^{\{k\}}, \phi_1^{\{k\}}, \phi_2^{\{k\}}, \dots, \phi_{n+1}^{\{k\}})\right\}_{k\in\mathbb{N}}$$

with some inintial guess for $k = 0$ by solving the problems

$$\boldsymbol{\mu}_0^{\{k+1\}}{}_\lambda = \operatorname*{arg\,inf}_{\boldsymbol{\mu}_0\in[\mathcal{M}(\Sigma_0)]^3} \mathcal{T}_{f,\lambda}(\boldsymbol{\mu}_0, \phi_1^{\{k\}}, \dots, \phi_{n+1}^{\{k\}})$$

$$(\phi_1^{\{k+1\}}, \phi_2^{\{k+1\}}, \dots, \phi_{n+1}^{\{k+1\}})_\lambda = \operatorname*{arg\,inf}_{(\phi_1,\dots,\phi_{n+1})\in D_2} \mathcal{T}_{f,\lambda}(\boldsymbol{\mu}_0^{\{k\}}, \phi_1, \dots, \phi_{n+1})$$

to achieve a minimiser of $\mathcal{T}_{f,\lambda}$. Using the results of [1, 2] we conclude in both the continuous and discrete versions of Problem 1 that the sequence generated above converges to a minimiser of $\mathcal{T}_{f,\lambda}$ with the functional $\mathcal{T}_{f,\lambda}$ converging to its minimum linearly.

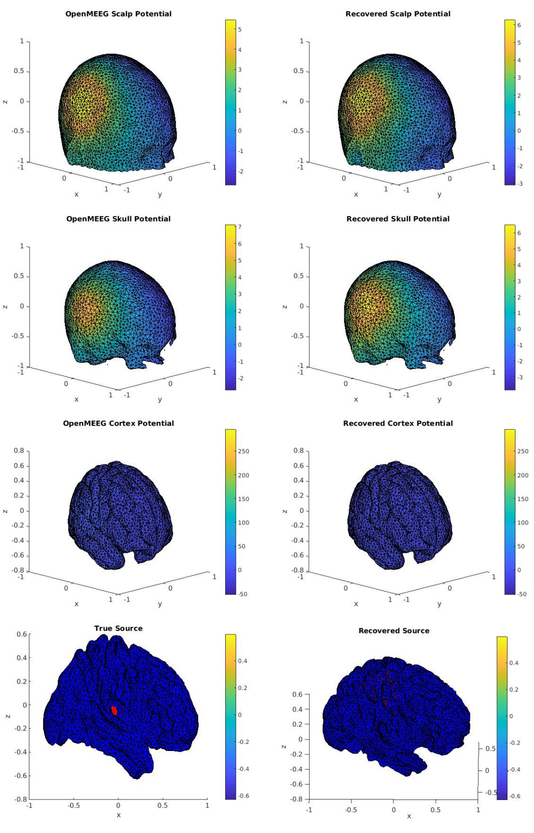

Figure 1: The synthetic data used here was generated in OpenMEEG. In these experiments we recover a dipolar source and perform the cortical mapping using the EEG data. The *possible* dipole locations are assumed *apriori* to be the barycentres of the triangles of the grey/white matter interface mesh. The red dots represent the dipole locations.

3 Numerical Experiments

We require triangular meshes of the grey/white matter interface, cortex, skull and scalp. Note that (1) and (2) contain boundary integral operators that are defined on the cortex, skull and scalp. From [3] these boundary integrals can be computed exactly for triangular meshes and piecewise linear ϕ's. When $\mathcal{M}(\Sigma_0)$ is the Banach space of Borel measures, we approximate $\boldsymbol{\mu}_0$ by a collection of dipoles. If the *possible* dipoles locations are *apriori* known and fixed, we use a FISTA algorithm with ℓ_1 penalisation to recover the dipole moments, see Figure 1 for EEG with $D_1 \subset \Sigma_{n+1}$. Whereas if the dipoles locations are allowed to be arbitrarily located on Σ_0, we use the algorithm in [4]. If $\mathcal{M}(\Sigma_0) = L^2(\Sigma_0)$ with $\boldsymbol{\mu}_0$ being normally oriented to Σ_0, we model the magnitude and orientation of the vector-field as a piecewise linear $L^2(\Sigma_0)$-function which results in Problem 1 being a least-squares problem. In this case a least-squares solution based on the Moore-Penrose pseudo-inverse can be obtained to solve the source recovery and cortical mapping problem together. The source is interpreted to be located in a neighbourhood of the extrema of the $L^2(\Sigma_0)$ function.

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