Modelling acoustic metasurfaces using homogenization of fluid-structure interaction on strongly heterogeneous perforated plates in thin layers

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Abstract

We consider acoustic waves propagating in a so-called transmission layer containing a perforated plate and inviscid fluid. The plate behavior is influenced by a strong heterogeneity of the elasticity in the form of periodically distributed holes and soft inclusions which may induce antiresonance effects well known in acoustic metamaterials. As a particular novelty, we study acoustic perturbations of the permanent flow interacting with the plate modeled using the Reissner-Mindlin theory. The modelling based on the homogenization leads to a 3D-to-2D model reduction of the layer which, in the limit, is represented by an acoustic metasurface. An efficient method is proposed to compute frequency-dependent homogenized coefficients involved in the limit model which are responsible for a strong wave dispersion.

Keywords: acoustic transmission, periodic homogenization, fluid-structure interaction, acoustic metasurface

1 Introduction

We consider a transmission layer $\Omega_\delta \subset \mathbb{R}^3$ of the thickness $\delta = \varkappa\varepsilon$ with a given fixed $\varkappa > 0$, being introduced via its midsurface Γ_0, see Fig. 1. The solid structure (an elastic perforated plate) Σ^ε is embedded in the layer, such that the acoustic fluid occupies domain $\Omega^{*\varepsilon} = \Omega_\delta \setminus \Sigma^\varepsilon$, The scale parameter $\varepsilon \sim \delta$ characterizes the microstructure sizes (holes and soft elastic inclusions, the resonators). whereby $\varepsilon = \delta/\varkappa$, with a given fixed $\varkappa > 0$.

The acoustic harmonic wave with the frequency ω propagating in the layer is described by the acoustic potential p^ε in the fluid $\Omega^{*\varepsilon}$, the corresponding wave in the elastic body is described by the displacement field $\boldsymbol{u}^\varepsilon$ defined in Σ^ε. These fields satisfy the following equations and transmission conditions on the solid-fluid interface $\partial_*\Sigma^\varepsilon$, where $\boldsymbol{\sigma}(\boldsymbol{u}^\varepsilon)$ is the stress,

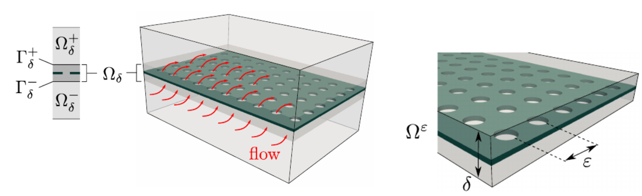

Figure 1: Thin layer with a perforated plate

$$
\begin{aligned}
c^2\nabla^2 p^\varepsilon + \omega^2 p^\varepsilon - (\theta \mathrm{i}\omega \partial_w p^\varepsilon + \tau \partial^2_{ww} p^\varepsilon) = 0 \quad & \text{in } \Omega^{*\varepsilon}\,, \\
\nabla \cdot \boldsymbol{\sigma}(\boldsymbol{u}^\varepsilon) + \omega^2 \rho \boldsymbol{u}^\varepsilon = 0 \quad & \text{in } \Sigma^\varepsilon\,, \\
\left.\begin{array}{rcl} \mathrm{i}\omega \boldsymbol{n}\cdot\boldsymbol{u}^\varepsilon & = & \boldsymbol{n}\cdot\nabla p^\varepsilon \\ \boldsymbol{n}\cdot\boldsymbol{\sigma}(\boldsymbol{u}^\varepsilon) & = & \mathrm{i}\omega\rho_0 p^\varepsilon \boldsymbol{n} \end{array}\right\} \quad & \text{on } \partial_*\Sigma^\varepsilon\,,
\end{aligned}
$$

where $\theta = (\tau+1)/2$, with $\tau = 3$ in 3D, and the derivatives $\partial_w p = \boldsymbol{w}\cdot\nabla p$ and $\partial^2_{ww} = \partial_w(\partial_w p)$ depend on the steady advection field$\boldsymbol{w}$.

The elastic structure is represented by the Reissner-Mindlin (RM) plate featured heterogeneity, such that it is generated by the representative cell $\Xi =]0,1[^2 \subset \mathbb{R}^2$ as a periodic lattice, where $\Xi = \Xi_S \cup \Xi^* \cup \partial\Xi$. The solid part involves the matrix and the soft inclusions, as treated in [1]. This latter aspect extends the work published in [2], such that vibroacoustic problem is featured by the large contrast in the differential operator associated with the RM plate equation. The weak formulation is constituted by the following variational equalities governing $p^\varepsilon \in H^1(\Omega^{*\varepsilon})$ and $(\boldsymbol{u}^\varepsilon, \boldsymbol{\theta}^\varepsilon) \in (H^1_0(\Omega))^5$,

$$
\begin{aligned}
& c^2 \int_{\Omega^{*\varepsilon}} \nabla p^\varepsilon \cdot \nabla q^\varepsilon - \omega^2 \int_{\Omega^{*\varepsilon}} p^\varepsilon q^\varepsilon \\
& + \mathrm{i}\omega\theta \int_{\Omega^{*\varepsilon}} (q^\varepsilon \partial_w p^\varepsilon - \partial_w q^\varepsilon p^\varepsilon) - \tau \int_{\Omega^{*\varepsilon}} \partial_w q^\varepsilon \partial_w p^\varepsilon = \\
& - \mathrm{i}\omega c^2 \left(\int_{\Gamma^{\pm\varepsilon}} g^{\varepsilon\pm} q^\varepsilon \, d\Gamma + \int_{\partial\Sigma^\varepsilon} \boldsymbol{n} \cdot \underbrace{\boldsymbol{U}^\varepsilon(\boldsymbol{u}^\varepsilon, \boldsymbol{\theta}^\varepsilon)}_{\text{R-M plate kin.}} q^\varepsilon \, d\Gamma \right),
\end{aligned}
$$

for all $q \in H^1(\Omega^{*\varepsilon})$, where $\boldsymbol{U}^\varepsilon = \overline{\boldsymbol{u}}^\varepsilon - \varepsilon\bar{h}\zeta\boldsymbol{\theta}^\varepsilon$, $\zeta \in [-1/2, +1/2]$ determines the displacement within the plate for the transversal position $\varepsilon\bar{h}\zeta$, being given by the plate kinematics defined in terms of the midsurface displacements $\boldsymbol{u} = (\overline{\boldsymbol{u}}, u_3)$, and rotations $\boldsymbol{\theta}$, satisfying

$$
\begin{aligned}
&\omega^2 \left(\int_{\Gamma^\varepsilon} \rho \boldsymbol{u}^\varepsilon \cdot \boldsymbol{v}^\varepsilon + \frac{h^2}{12} \int_{\Gamma^\varepsilon} \rho \boldsymbol{\theta}^\varepsilon \cdot \boldsymbol{\psi}^\varepsilon \right) \\
&- \frac{h^2}{12} \int_{\Gamma^\varepsilon} [\mathbb{E}^\varepsilon \overline{\nabla}^S \boldsymbol{\theta}^\varepsilon] : \overline{\nabla}^S \boldsymbol{\psi}^\varepsilon - \int_{\Gamma^\varepsilon} [\mathbb{E}^\varepsilon \overline{\nabla}^S \overline{\boldsymbol{u}}^\varepsilon] : \overline{\nabla}^S \overline{\boldsymbol{v}}^\varepsilon \\
&- \int_{\Gamma^\varepsilon} [\boldsymbol{S}^\varepsilon (\overline{\nabla} u_3^\varepsilon - \boldsymbol{\theta}^\varepsilon)] \cdot (\overline{\nabla} v_3^\varepsilon - \boldsymbol{\psi}^\varepsilon) \\
&= \frac{1}{\varepsilon \bar{h}} \int_{\Gamma^\varepsilon} (\boldsymbol{f}^\varepsilon(p^\varepsilon) \cdot \boldsymbol{v}^\varepsilon + \overline{\boldsymbol{m}}^\varepsilon(p^\varepsilon) \cdot \boldsymbol{\psi}^\varepsilon) \\
&\frac{1}{\varepsilon \bar{h}} \int_{\partial_\circ \Gamma^\varepsilon} \left(\overline{\boldsymbol{f}}^{\partial,\varepsilon}(p^\varepsilon) \cdot \overline{\boldsymbol{v}}^\varepsilon + \int_{\partial_\circ \Gamma^\varepsilon} \overline{\boldsymbol{m}}^{\partial,\varepsilon}(p^\varepsilon) \cdot \boldsymbol{\psi}^\varepsilon \right) ,
\end{aligned}
$$

for all $(\boldsymbol{v}^\varepsilon, \boldsymbol{\psi}^\varepsilon) \in (H_0^1(\Omega))^5$. The r.h.s. terms represent plate loading by p^ε on its surface.

2 Homogenized transmission layer

Pursuing analogical upscaling procedure based on the unfolding homogenization, as reported in [2], autonomous cell problems defined in Y^* and Ξ_S, see Fig. 2, are solved for characteristic responses. These are needed to establish the macroscopic problem involving homogenized coefficients. In the context of a global acoustic problem, the homogenized layer model presents a Dirichlet-to-Neumann operator which links the acoustic fluxes $g^0 \approx \partial_n \hat{P}$ to the the pressure jump $\Delta \hat{P}$, thus representing transmission conditions on Γ_0, which involve internal variables $(p^0, \boldsymbol{u}^0, \boldsymbol{\theta}^0)$ satisfying the fluid equation,

$$
\begin{aligned}
&\int_{\Gamma_0} \left[(\boldsymbol{A} \overline{\nabla}_x p^0 + \mathrm{i}\omega g^0 \boldsymbol{B}_w) \cdot \overline{\nabla}_x q^0 - \omega^2 (\zeta^* + M_w) p^0 q^0 \right] \\
&+ \mathrm{i}\omega\theta \int_{\Gamma_0} q^0 \left(\mathrm{i}\omega T_w g^0 + \bar{\boldsymbol{W}} \cdot \overline{\nabla}_x p^0 \right) + \mathrm{i}\omega \int_{\Gamma_0} q^0 \Delta G^1 \\
&+ \mathrm{i}\omega \int_{\Gamma_0} \left(q^0 \bar{h} \boldsymbol{H} : \overline{\nabla}_x^S \overline{\boldsymbol{u}}^0 + \overline{\nabla}_x q^0 \cdot \boldsymbol{D} \boldsymbol{u}^0 \right) = 0 ,
\end{aligned}
$$

$\forall q^0 \in H^1(\Gamma_0)$, the plate equation

$$
\begin{aligned}
&\int_{\Gamma_0} (\boldsymbol{S}^{\mathrm{H}} (\overline{\nabla}_x u_3^0 - \boldsymbol{\theta}^0)) \cdot (\overline{\nabla}_x v_3 - \boldsymbol{\vartheta}) \\
&+ \frac{h^2}{12} \int_{\Gamma_0} (\mathbb{E}^{\mathrm{H}} \overline{\nabla}_x^S \boldsymbol{\theta}^0) : \overline{\nabla}_x^S \boldsymbol{\vartheta} + \int_{\Gamma_0} (\mathbb{E}^{\mathrm{H}} \overline{\nabla}_x^S \overline{\boldsymbol{u}}^0) : \overline{\nabla}_x^S \overline{\boldsymbol{v}} \\
&- \mathrm{i}\omega \rho_0 \int_{\Gamma_0} \left[p^0 \boldsymbol{H} : \overline{\nabla}_x^S \overline{\boldsymbol{v}} + \frac{1}{\bar{h}} \boldsymbol{v} \cdot \left(\boldsymbol{D} \overline{\nabla} p^0 + \mathrm{i}\omega \boldsymbol{C} g^0 \right) \right] \\
&- \omega^2 \int_{\Gamma_0} \left((\boldsymbol{M} \boldsymbol{u}^0) \cdot \boldsymbol{v} + \frac{h^2}{12} (\tilde{\boldsymbol{\mathcal{M}}} \boldsymbol{\theta}^0) \cdot \boldsymbol{\vartheta} \right) = 0 ,
\end{aligned}
$$

satisfied $\forall (\boldsymbol{v}, \boldsymbol{\vartheta}) \in (H_0^1(\Gamma_0))^5$ and a coupling condition (for a given scale $\varepsilon_0 > 0$)

$$
\int_{\Gamma_0} \psi \left(\mathrm{i}\omega \boldsymbol{C} \cdot \boldsymbol{u}^0 - \mathrm{i}\omega F g^0 \right) = \frac{1}{\varepsilon_0} \int_{\Gamma_0} \psi \Delta \hat{P}
$$

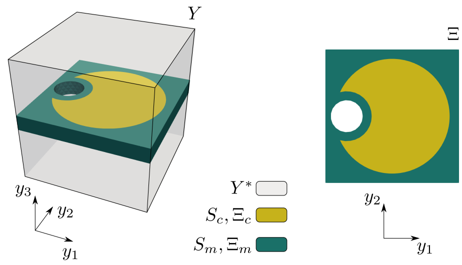

Figure 2: Representative cell $Y \subset \mathbb{R}^3$: Fluid part $Y^* = Y \setminus S$. The solid part S represented by the plate $\Xi_S \subset \Xi \subset \mathbb{R}^2$ with soft inclusions Ξ_c and a stiffer "matrix" Ξ_m, $\Xi_S = \Xi_m \cup \Xi_c$.

$\forall \psi \in L^2(\Gamma_0)$, which closes the system. Due to the strong heterogeneity of the plate [1], the derived macroscopic model of the layer represented by midsurface Γ_0 involves ω-dependent coefficients which can change their signs, thus, giving rise some very strong dispersion property of the "metasurface". This is illustrated in Fig. 3 showing acoustic fields in a waveguide for two frequencies ω_1, ω_2. For ω_2 the antiresonance effect suppress significantly wave propagation through the perforated plate. *Research supported by project GACR 21-16406S.*

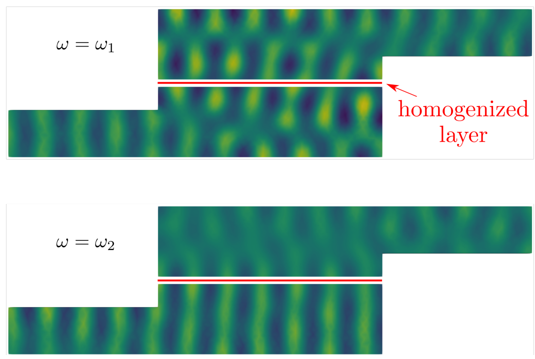

Figure 3: Effects of suppressing the acoustic field for some frequencies (here for ω_2) inducing the metasurface behaviour.

References

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