Elasticity, skeletal muscle, and waves?

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Abstract

In this talk, we survey recent work on the modeling, simulation and validation of a fully 3-D continuum elasticity approach for skeletal muscle dynamics. Skeletal muscle is modelled as a fibre-reinforced hyperelastic material, with other connective tissues such as aponeurosis and tendon being similarly described. These fibres are capable of nonlinear *activation*. After discretization (semi-implicit in time, FE in space), the model is validated against physiological data, and then used to understand the impact of muscle architecture, mass and tissue properties on questions of physiological interest.

Keywords: Nonlinear elasticity, skeletal muscle mechanics, three-field formulation.

1 Background

Skeletal muscles exhibit fascinating structural and mechanical properties. Skeletal muscle is composed of cells collectively referred to as *fibers*, which themselves contain contractile proteins arranged longtitudinally into *sarcomeres* (Fig.1). These latter respond to signals from the nervous system, and contract; this leads to a strong mechanical anisotropy in the system. Muscles react to mechanical forces - they contain connective tissue and fluid, and are linked via tendons to the skeletal sytem - but they also are capable of *activation* via stimulation (and hence, contraction) of the sacromeres . The restorative along-fibre force depend on departures from a characteristic length of the sarcomeres; diseases such as cerebral palsy cause this characteristic length to change, thereby impacting muscle force.

Prior to the landmark paper by A.V. Hill [1], it was believed a stimulated muscle was like an elongated spring that has the capacity to contract and do work. However, this failed to explain an important distinction between 'usual' elastic materials and skeletal muscle: the force exerted by a pure elastic body depends on its strain; however, in muscle fibres, force is additionally dependent on the velocity of contraction. Hill's paper showed that even for *isometric* (fixed length) contractions, muscles fibres are capable of shortening. Hill suggested that skeletal muscles have two distinct kinds of elastic components in series with each other: a contractile component that shortens when stimulated and a nonlinear elastic component which lengthens under tension. The resulting 1-D mathematical model (see Fig.2) proposed by him was both simple and remarkable in its predictive capabilities. Since then, experimentalists have gained much insight into mammalian skeletal muscle especially at small scales such as those of sarcomeres, single fibres and small muscles. Experimental data on muscle contraction is typically determined assuming the muscle is fully active, changes length at constant velocity, and considers forces and length changes in only the longitudinal direction. This information, incorporated into refinements of Hill's model, has lead to important advances in biomechanics.

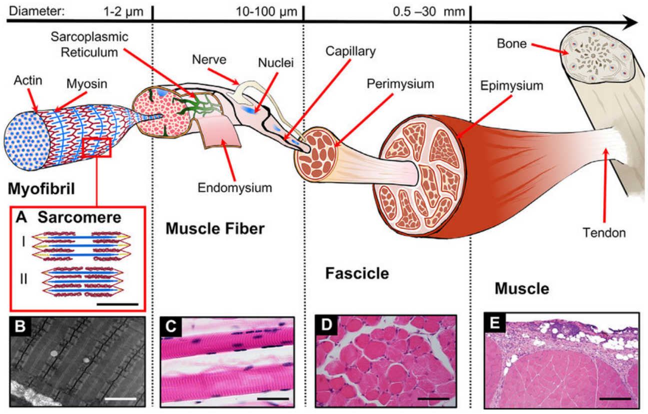

Figure 1: Hierarchical depiction of muscle, [13]. We study the elastic response of muscle-connective tissue complex at the mm-cm scale.

In Hill's 3-element model Fig 2, a muscle fibre of total length L is described as $L = L_{PE} = L_{SE} + L_{CE}$, where L_{PE}, L_{CE} and L_{CE} are the lengths of the passive, contractile and series elements. The force in the contractile element depends on both the stretch λ and the time rate

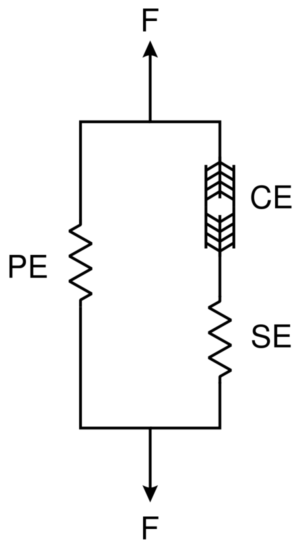

Figure 2: Hill's 3-element model: PE denotes the *passive* element, CE is the (nonlinear) *contractile element*, SE encapsulates the elastic properties of the fibre.

of stretch $\dot{\lambda}$:

$$F_{CE}(L_{CE}, v) = \underbrace{F_{max}\left[a(t)\hat{F}_v(\dot{\lambda})\hat{F}_a(\lambda)\right]}_{Active} + \underbrace{F_{max}\hat{F}_p(\lambda)}_{Passive}.$$

The springs themselves are assumed massless, and to move in 1 direction.

Skeletal muscles consist of many fibres, arranged in a *striated* manner (in contrast to cardiac muscle). We know surprisingly little about large muscles contract, particularly when they are not fully active or contract with varying velocities. We also do not know, in detail, what the effects of changing shape, muscle density or material properties (fat infiltration, stiffening due to neuromuscular diseases) have on muscle force output. Understanding how the contractile elements interact with the tissue properties of the whole muscle, how deformations may arise in all three dimensions during contraction, and how transverse compression/shape changes affect force output are questions which cannot be answered by 1-D models. For example, experimental evidence suggests that the Hill-type approximation yields poor force predictions for larger muscles, [11]. Additionally, it has been experimentally observed that elastic waves may propagate through the muscle; this is not possible to explain the single-fibre model of muscle.

Some of the pioneering works on a fully 3-D model of muscle include [4,6,7], who considered a 3-D constitutive model for incompressible biological soft tissues within an isometric setting, and FEM implementations. In this talk, we'll first present a brief review of the existing work on 3-D modelling of skeletal muscle mechanics. We'll describe our model, and some of the experimental data used to fit parameters. The discretization of the highly nonlinear system is via a semi-discretization in time, and a finite element discretization in space.

2 Mathematical model

We focus on length scales larger than those of individual fibres, which allows us to capture the role of structure and tissue properties on large muscle mechanics. Muscle is represented as a fiber-reinforced hyperelastic material, where the fibre properties are governed by the myofilament contractile forces described Hill-type models, and the composite properties are represented a Neo-Hookean 'base material' encapsulating properties of muscle tissue across several scales, including intracellular stiffness and extracellular material. This muscle is encased, as needed, by connective tissues (tendon/aponeurosis) modelled as Yeoh-type materials.

Our work builds on that of [6, 7]. We work with a three-field variational formulation pioneered in [2, 3] (see also [5]). The implementation is within the `deal.ii` finite element library, and is based on the excellent tutorial on finite deformations in an isotropic Neo-Hookean material. [8].

We first discuss quasi-static deformations, consistent with an isometric system in which neither muscle mass nor velocity is involved. We use a mixed Jacobian formulation ([2,3])to solve for the unknown displacement $\mathbf{u}$, the pressure p, and a dilation D in the current configuration Ω, with $\Pi := (u, p, D) \in (H^1(\Omega))^3 \times L^2(\Omega) \times L^2(\Omega)$. We seek the state Π which is a stationary point of a potential (as in [8]).

The constitutive relations for hyperelastic materials are given, as is standard, in terms of the Helmholtz free energy density $W(\mathcal{B})$, which depends on the left Cauchy-Green tensor $\mathcal{B}$. We can split the energy density into a volumetric and isochoric part; the latter is then split into the along-fibre and base material contributions

$$W(\mathcal{B}) = W_{vol}(J) + W_{iso}(\bar{\mathcal{B}}) = W_{vol}(J) + W_{fibre} + W_{base}.$$

See, for instance, [6,12]. The precise dependance of W on different invariants of $\mathcal{B}$ are obtained via fitting to experimental data; details can be found in [9, 10, 12, 14, 15]. As an instance: in order to obtain the energy W_{fibre} (which is consistent with the original Hill-type model), we consider single-fibre data. In the laboratory setting, what is measured is the dependance of along-fibre stress with isochoric stretch λ_{iso}. So, we must first fit the stress to the data (Fig 3 [11]; we'd then use the relation

$$\lambda_{iso}\frac{\partial W_{fibre}(\lambda_{iso})}{\partial \lambda_{iso}} = \sigma_{fibre}(\lambda)$$

to obtain W_{fibre}. A similar procedure is carried out for the other components of the model. In some, the experimental data is directly in terms of the strain energy; in others (as for the fibre), it is not.

The Euler-Lagrange equations for stationarity of the potential for the isometric setting can then be written as

$$-div(\sigma(\mathbf{u})) = \quad \mathbf{b} \quad \text{(static equilibrium)} \tag{1}$$

$$D = \quad det(\mathcal{I} + \nabla \mathbf{u}) =: J(\mathbf{u}) \quad \text{(dilation)} \tag{2}$$

$$p = \quad \frac{\delta(W_{vol(D)})}{\delta D} \quad \text{(pressure response)} \tag{3}$$

We prescribe either zero traction boundary conditions on the faces, or Dirichlet conditions (depending on the experiment). We also allow for the combination of distinct tissues - muscles, aponeurosis and tendon - for which the constitutive laws have to be obtained.

As in [8], we use a total Lagrangian formulation. The Euler-Lagrange equations leads to a (nonlinear) weak formulation. We use a finite element approach, combined with a Newton-Raphson strategy to solve the nonlinear system; the system is implemented within the `deal.ii` library.

In case of a fully dynamic system, inertial effects due to mass become important. In addition, the along-fibre contributions to the stress are given by a (nonlinear) relationship involving both the stretch and the stretch rate; this is due to Hill's model. For this reason, we must directly work with the dynamic system with the

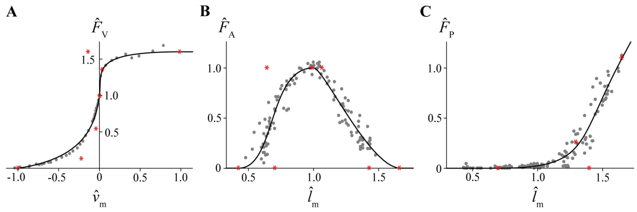

Figure 3: From [11]: Bezier-curve fits to single-fibre data. Left: Force- velocity $\hat{F_v}$, (M): Active force-length $\hat{Fa}$, R: Passive force-length $\hat{F_p}$.

additional unknown velocity $\mathbf{v}$.

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \mathbf{v}) &= -div(\rho \mathbf{v} \otimes \mathbf{v}) + div(\sigma(\mathbf{u})) + \mathbf{b}, \\ D &= J(\mathbf{u}), \\ p &= \frac{\delta(W_{vol})}{\delta D}. \end{aligned}$$

Once again, zero traction and (possible inhomogenous) Dirichlet conditions are allowed. We use a semi-implicit discretization in time, and a $Q1 \times P0 \times P0$ discretization in space.

3 Results

Our mathematical models, and associated finite element implementations, allow for an exploration of a range of questions in physiology. A first and important question to address is: in pennate muscles (those in which the line-of-action of a load is not long the direction of fibres), does the *curvature* of fibres change? This has been observed in MRI studies, and cannot be readily explained by 1-D Hill-type models.

As a next investigation, we examine the mechanical energy within a muscle. During muscle contraction, chemical energy is converted to mechanical energy, which in turn is distributed and stored in the tissue as the muscle deforms or is used to perform external work. We showed how energy is distributed through contracting muscle during fixed-end contractions; subsequently, we study the distribution of tissue energy when mass effects are taken into account. Some of these results will be described in the talk, and also form the basis of *validation* of our model, [12, 14, 15].

As a final demonstration, we will describe recent work on cerebral palsy. In this condition,there are changes to the size, shape and stiffness of the tissues; additionally, the length of the sacromeres changes (resulting in changes to the curves in Fig. 2). MRI data is used to generate hexahedral meshes, and we are able to assess the impact of these changes to the mechanical work done by muscles. [16].

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